United Kingdom
Mathematics Trust

Intermediate Mathematical Olympiad

Maclaurin paper

Thursday 16 March 2023

England & Wales: Year 11
Scotland: S4
Northern Ireland: Year 12

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: 2 hours.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; squared paper, calculators and protractors are forbidden.
4. Start each question on an official answer sheet on which there is a QR code.
5. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. Please do not write your name or initials on additional sheets. **Do not hand in rough work.**
6. Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

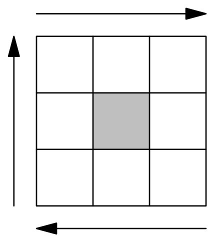
challenges@ukmt.org.uk

www.ukmt.org.uk

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but* not *decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do* not *hand in rough work.*

1. A plank of wood has one end, A, against a vertical wall. Its other end, B, is on horizontal ground. When end A slips down 8cm, end B moves 4cm further away from the wall. When end A slips down a further 9cm, end B moves a further 3cm away from the wall. Find the length of the plank.

2. The digits 1 to 8 are placed into the cells of the grid on the right, making four three-digit numbers when read clockwise. For which values of k from 2 to 6 is it possible to create an arrangement such that all four of the three-digit numbers are multiples of k?



3. $ABCD$ is a square and X is a point on the side DA such that the semicircle with diameter CX touches the side AB. Find the ratio $AX : XD$.

4. The ratio of the number of red beads on a chain to the number of yellow beads is the same as the ratio of the number of yellow beads to the number of blue beads. There are 30 more blue beads than red ones. How many red beads could be on the chain?

5. A 4 by 4 square is divided into sixteen unit cells. Each unit cell is coloured with one of four available colours, red, blue, green or yellow.
The 4 by 4 square contains nine different 2 by 2 “sub-squares". Suppose that we colour the sixteen unit cells in such a way that each 2 by 2 sub-square has one cell of each colour.
Prove that the four corner cells in the large 4 by 4 square must then be coloured differently.

6. Let m, n be fixed positive integers. Prove that there are infinitely many triples of positive integers (x, y, z) such that

$$x^{mn+1} = y^m + z^n$$

for each pair of values (m, n).